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INSTRUMENTALIZATION OF ORIGAMI IN CONSTRUCTION OF FOLDED PLATE STRUCTURES - DESIGN, RESEARCH AND EDUCATION

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The paper deals with the origami used as an abstract tool to describe and represent the form and the structure of physical objects. In that respect, the potentials of this interdisciplinary technique as a medium of exploration of structural forms was introduced in the semester project done within the course Structural Systems at the Belgrade University, Faculty of Architecture. The technique was used as an interface to gain cognitive experience on spatial transformation and computational design. Throughout the intensive project period divided into three successive stages, the objective was to test method which enabled students to analyze geometrical principles of folding in order to apply these principles in the development of new designs. The generative algorithm inspired by the technique of paper folding assisted form-finding. Resulting shapes were verified by a production of small scale prototype models. The applied method, as a guiding design principle, facilitated formal exploration and augmentation of the design process. At the end of the course, students got cognitive experience on structural forms, while this simple technique delivered richness in terms of design solutions.

Key words: **origami, folded-plate structures, form-finding, computer aid geometric modeling, rapid prototyping.**

INTRODUCTION

The rapid evolution of technology and increase in complexity are continuously changing the design environment. The challenges we face involve the convergence of design process driven by the proliferation of the CAD/CAM systems and automated production processes. These technologies enable experiments based on computational generation and digital fabrication of structures unconventional in their form, typology, and aesthetics.

The increase in computational capabilities led to projects being conceived and elaborated in an integrated context in which architecture and other fields continuously exchange, analyze and produce diverse information. The expression of this condition is exemplified with design experiments in which architects practice discourses often borrowed from other disciplines. As the strict boundaries between disciplines are increasingly questioned and broadened, technology development and application, as well as incorporation of overlapping patterns, emerge as an essential vehicle for design exploration.

We approached the issue of the design of folded-plate systems by applying origami techniques in order to create proposals for steering design. Origami has evolved from being a craft to an interdisciplinary method *Origamics* (Stewart, 2007). With a wide range of applications origami proved itself as an advantageous tool for the development of different engineering and design solutions. During the last decades, trans-disciplinary studies in the fields of biology, nanotechnologies and automotive and structural design offered possibilities for the exploration of geometrical relations, new forms and structures. The amazing technical and artistic advancements, realized largely due to a growing mathematical and computational understanding and analysis of the subject (Hagiwara, 2008), and the increasing number of new examples, researches and exhibitions demonstrate that this ancient technique still has many prospects to be explored.

While the advancement of information theories and technologies found its interpretation in the design of

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architectural systems, leading architects to explore new form-function-structure relations, design problems became increasingly complex. Promoting development of new methods, media and tools that will facilitate such inquires became a very important issue. In that regard, this paper reviews sustainability of origami as a medium of inquiry into the design of folded-plate systems. Additionally, the paper indicates how origami could be of service for architectural design education as a potential interface to gain cognitive experience on spatial transformations, computational design, and shape-finding of structural forms. We tested the effectiveness of this approach through the semester project introduced in the course Structural Systems at the Belgrade University, Faculty of Architecture.

STATE OF ART

The idea to increase structural heights of sections by folding, and thus improve overall stiffness, was not applied before the time of the development of reinforced concrete in the XX century. The first hint of a folded-plate system was a hangar in Orly designed by Freyssinet in 1916 (Nestorović, 2000). The explanation of the static behavior and the studies on shaping possibilities implied wider application of folded-plate systems. Besides scientific researches done in the early 1950s, Moretti conducted experiments with paper models that were later continued by Flayshman. The results of tests confirmed great stiffness, structural fitness and feasibility of the system (ibid.). It should be pointed out that in folded surface systems, form, structural behavior, and construction techniques are interrelated. In order to sustain this consistency, the structural considerations should be regarded from the phase of design conception. Respectively, architects exploiting the folded structures need to have a fundamental understanding of the behavior of the rigid surfaces.

Structural System

Folded plates are thin structural surfaces that can achieve large spans due to the stiffening effect of folding. Even the simplest folded-plate system, consisting of two planar plates joined at a single fold line, when the ends are secured with rigid diaphragms, is remarkably stiff. The simple folded-plate can easily be arrayed to produce corrugated surface (Bechthold, 2008).

The system has two-way behaviors. In the direction of the primary span, the system behaves beam-like and approximate bending and shearing stresses can be derived in much the same manner as for ordinary beams. In the direction perpendicular to the primary span plates behave like slabs or members in bending, with each fold acting as support. By providing a rigid connection between plates at the fold strips, the system can be understood as a continuous beam or slabs with supports at the fold lines (Nestorović, 2000).

The estimation of the system's stability is not always straightforward, especially in the case of complex assemblies. According to Bechthold (2008), one of the easiest ways to determine stability is by considering the elevation view parallel to primary span and checking for locations where the depth of the overall system is reduced to plate thickness. Because the out-of-plan bending strength of the plate is insufficient and unable to resist deformations, these areas will behave as hinges. The hinges could be avoided by providing adjacent folded plates necessary structural depth. Systems like three-hinge frames or arches are statically determinate and more relaxed with respect to differential support settlements and thermally induced stresses. Clearly, having more than three hinges any one-way will lead structural system to instability.

A variety of arrangements that use both triangular and quadrilateral plates could be applied to produce a range of solutions from curved, arch-, vault-, frame-like systems to continuous beams. Identically to the equivalent linear or curved discrete systems, arches, vaults and frames composed of folded-plate elements develop both horizontal and vertical reactions. Also, the formation of openings in the folded-plate system should be located exclusively in zones of moderated bending moments and shears, generally near the neutral axis or plane. Creation of closely spaced large openings might change the nature of the structural system from a plate to a Virandeel truss or frame. Another possibility, as seen in the Yokohama Ferry Terminal designed by Foreign Office Architects, is the replacement of plates by triangulated trusses (Bechthold, 2008). In this case the combination of trusses with thin steel sheets creates a hybrid shear resistant system that is equivalent to a plate.

Origami as Architectural Design Methodology

The contemporary use of origami is not restricted to craft. At the same time it could serve as a problem-solving method, as a tool for education, or for the purpose of diverse design applications. While, traditionally, origami was designed through heuristic techniques based on folder's instincts, understanding mathematical and geometrical pattern relations are essential parts of current studies (Nestorović *et al.*, 2012). In the context of mathematics, origami forms can be considered as the mapping of tessellations into the 2D and 3D space (Sorguç *et al.*, 2009). In general, these tessellations are divided from applying isometric and/or similarity transformations of lines and line shapes in the 2D space.

The development of most of origami patterns it established on a *grammar,* consisting of shapes (lines and angles), and *grammar rules*. Mostly, the geometry is based on Huzita's Axioms (Khademzadeh and Mazaheri, 2007), Maekawa's Fundamental Theorems (Maekawa, 2008), Muira's Patterns (Miura, 1994, 1997), Kawasaki Theorems (Hull, 2002), as well as theorems and axioms proposed by many other mathematicians. Lang was the first to convert these axioms and theorems into algorithms that contributed to the achievements of more complex origami forms and development of commercial software (Lang, 1994, 1996, 2004).

In the quest for both aesthetically appealing and structurally efficient solutions, architects have been searching for new form-structural relations and exploring new design tools. Form-structural duality of origami that enables control of spatial relations recommends this technique as possible design tool to facilitate that search. Numerous researches on application of origami in structural design, for example, studies of kinetic and deployable structures (Ebara and Kawaguchi, 2003) or biomimetic researches (Vincent, 2000; Hachem *et al.*, 2004; Hagiwara, 2008; Kobayashi *et al.*, 1998), confirm potentials of this technique.

In architectural practice origami usually finds application in the design of surfaces and kinetic structures. Besides mentioned Yokohama Port Terminal (Moussavi, 2009), the Colorado Springs Air Force Academy Chapel by SOM represents another well-known example of folded-plate building in which the diagrams and structural relations of origami can be traced easily (SOM, 2011), as well as the folded dome of the Assembly Hall of the University of Illinois designed by Abramovitz, or the roof of the American Institute for Concrete in Detroit by Yamasaki (Nestorović, 2000). Another example, the Chapel in St Loup, designed by Local architecture and Mondana in collaboration with the IBOIS, demonstrates the feasibility of the application of scientific researches in design practice (Buri and Weinand, 2008).

Origami could be applied in the design of deployable structures. Diagrams developed in origami can be considered as the relation of *links* which are designed to yield either a translational or rotational displacements without locking problems. Even complex origami folding diagrams can be easily folded and unfolded with a single stroke of a force in a stable way, exemplified by *folding egg* prototype constructed from a low-cost recyclable material by B. Yeh form Kinetic Design Group at the MIT (Sorguç *et al*., 2009).

Material and Construction Techniques

The thinness and geometry of folded-plate structures represent a challenge for construction, making traditional methods and techniques often inadequate. The production of complex geometries has always been difficult for an industry attuned to orthogonal forms. Economic needs often limit construction and consequently the design of intricately folded surfaces imposing multiplication of identical elements.

The development of digital design and fabrication techniques are opening new opportunities for construction of folded shapes. A wide range of approaches includes the use of prefabricated formwork elements, 3D digital models in the prefabrication of elements, the application of high-speed CNC for both shaping of elements and formwork, as well as origami-like strategies to create overall complexity using simple, planar elements that can be easily varied in size (Bechthold, 2008). There are tendencies to move construction in a controlled prefabrication environment more favorable for digital production, as well as studies on the on-site automated construction.

Origami has a potential to provide a prototype and construction algorithm as well. The example is the idea of *folding concrete*, patented by Wheen (1980). The feasibility of this idea was tested through the fabrication of large scale prototype *concrete origami* at the Harvard University Graduate School of Design (Bechthold, 2008). In a study done by Banghay (2000), origami was exploited to help the decomposition of virtually constructed objects for the CAM manufacturing. Furthermore, aid in reverse engineering represents another possible field of application.

The development of high-performance materials also brings prospect in the design and construction of folded-plate system. Glass technology, ultra-high-strength fiber concrete, and polymer composites are starting to be introduced in construction. However, the first schemes are promising and will surely lead to new developments in the folded-plate architecture extending pallet of currently applied materials. On the other hand, origami also inspired methods used in the development of nano-materials (Stellman *et al*., 2005), that in the future might find application in the building industry.

DESIGN RESEARCH

A very direct and spontaneous perception and comprehension of geometry and rigidity of folded plate structures can be achieved simply by folding paper. Correspondingly, the technique has always been inspiring for the application in educational processes. Fröebel used paper folding in kindergartens to teach geometry and promote the sense of aesthetics in his pupils (Lister, 2003, 2004). Albers used similar methods in the preparation class for the Bauhaus to make his students discover the relationship between materiality, geometry and structure (Albers, 1952). Currently, there are numerous courses at different architectural schools and workshops which allow students to explore these form-structure-space concepts both in real and virtual context. Driven by the similar motivations we introduced this method in the semester project at the second year course Structural Systems at the University of Belgrade, Faculty of Architecture.

Project Procedure

Throughout the intensive project period, the objective was to test the method, which enabled students to analyze geometrical principles of folded-plate structures and to apply them in the development of new designs (Nestorović *et al*., 2012) (Figure 1).

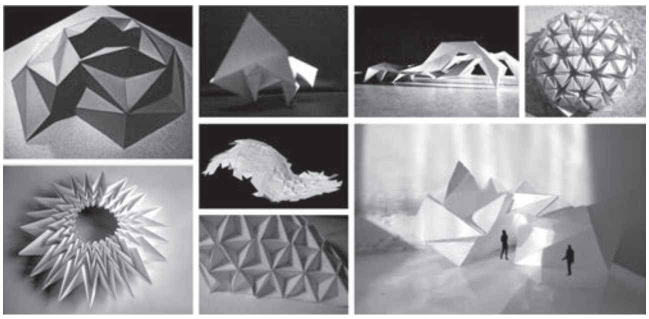

Figure 1. Design concepts proposed by the students of the University of Belgrade, Faculty of Architecture
(Source: processed by authors)

In order to strengthen the design process the project was divided into three successive phases: analysis, transformation and elaboration, where each stage was built upon the results of the previous one (Figure 2). In order to obtain the overall picture, the phases of the procedure will be illustrated using one example of the folded-plate design solution for the pavilion.

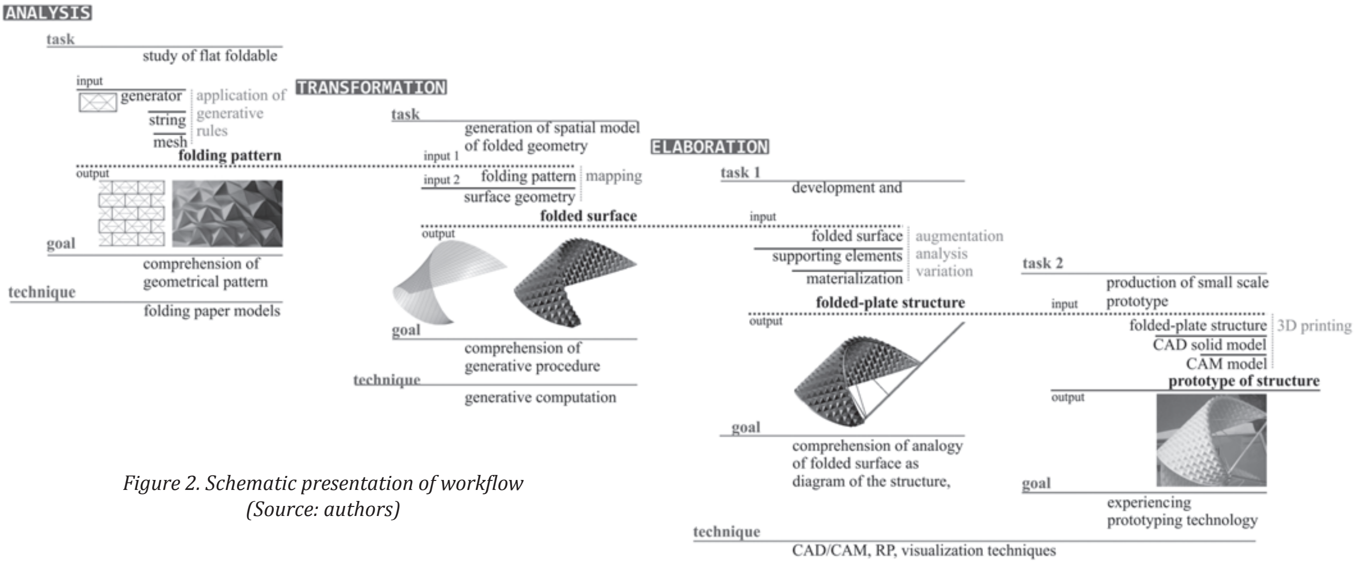

Figure 2. Schematic presentation of workflow
(Source: authors)

Analysis

During the first stage geometrical and structural principles were analyzed. Students divided in groups were tasked with studying diverse foldable tessellations by making physical models. The production of a series of folded paper models enabled the exploration of formal and spatial advantages of the technique and its application in the context of architectural design. The goal of this part of the project was to identify some interesting folding patterns that have potential to be implemented in the design. In that respect, different folding patterns, tempting for architectural application, were exploited, such as Yoshimura pattern (Diamond Pattern), Miura Ori pattern (Herringbone Pattern), Diagonal pattern, patterns with polygons, etc. (Jackson, 2011). These patterns are mostly based on a combination of simple folding and reverse folding techniques, in which a series of straight valley and mountain folds are bent by the reverse folds to form corrugated surfaces.

Understanding mathematical and geometrical relations were an essential part of these studies, starting with the selection of fundamental units of patterns – generators. The application of diverse generative rules on generators resulted in the production of strings and mashes. Meshes were then translated into folding patterns, i.e. diagrams which indicated mountain folds, valley folds and flat folds (Figure 3).

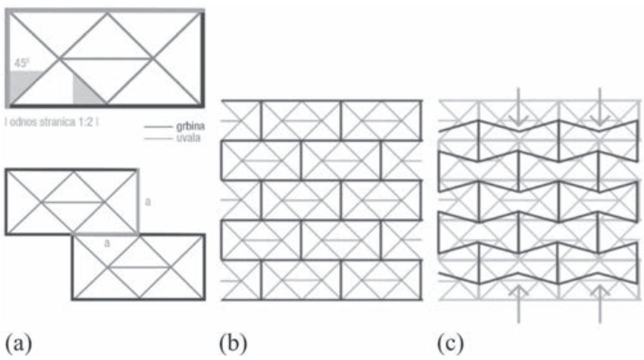

Figure 3. Generator (a), mash (b), folding pattern (c)
(Source: authors)

Manipulation of generic parameters and generative rules resulted in creation of folds (Figure 4).

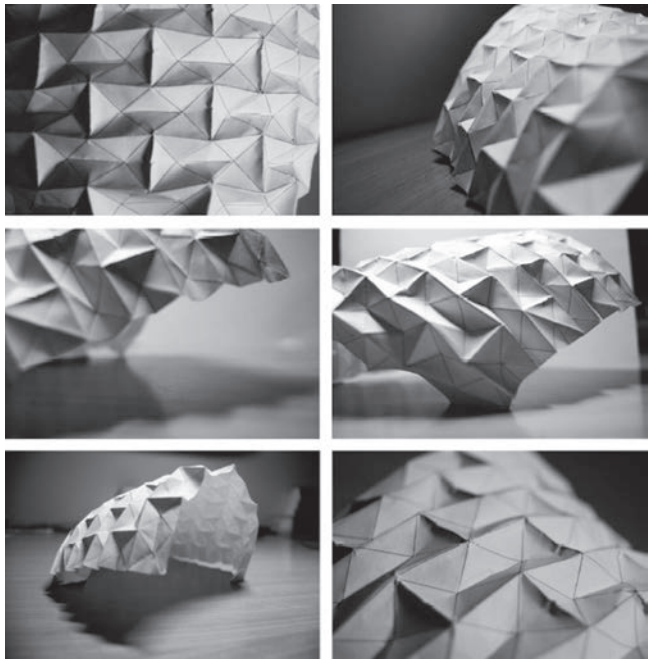

Figure 4. Folded paper models
(Source: processed by authors)

Transformation

In the phase of transformation the selected folding patterns were mapped on defined architectural geometries in order to form models of space structure. With the purpose of creating diverse architectural shapes by using folds, classes of geometric types (Pottman *et al*., 2007) and their compositions were used as an underlying geometry. This part of the study was aimed at the analytical understanding of the chosen geometries which resulted in their generation by the CAD software. Exploring the potentials of origami as an algorithmic tool for form-finding was main objective of this phase.

The mapping of the folding pattern on defined reference geometry was implemented by exploiting the concept of hinge lines (Mitra, 2009). Hinge lines connect nodal points in which four or more fold lines meet and define and control the degrees of freedom (DOF) that fold surfaces geometrically possess. By operating with their length and shape in the 3D space it is possible to define specific fold configuration. Surface geometries could be approximated by grids of lines as wire models. Correspondingly folded surfaces can be represented by a grid of control lines that govern them. The advantage of these properties was used in the mapping procedure (Figure 5).

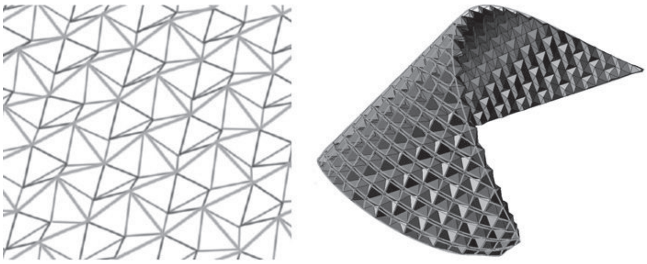

Figure 5. Mapping of fold tessellation on architectural geometry (Source: processed by authors)

Studies showed that some folds had more DOFs than others, and could fit varied forms. Also, though they were generated using same or similar generators, certain folds were ideal for particular forms and not the others (Nestorović *et al*., 2012). Additionally, applied generative method allowed rapid creation of different forms that can adapt to specific project conditions.

Elaboration

In this phase, based on the geometry model of folded surface, the design solution for folded-plate structure was elaborated and presented by the application of diverse techniques. The focus was on the relations between form-structure and construction-materialization. The goal was the transformation of folded surface diagram in real rigid structure composed of appropriate structural elements (Figure 6).

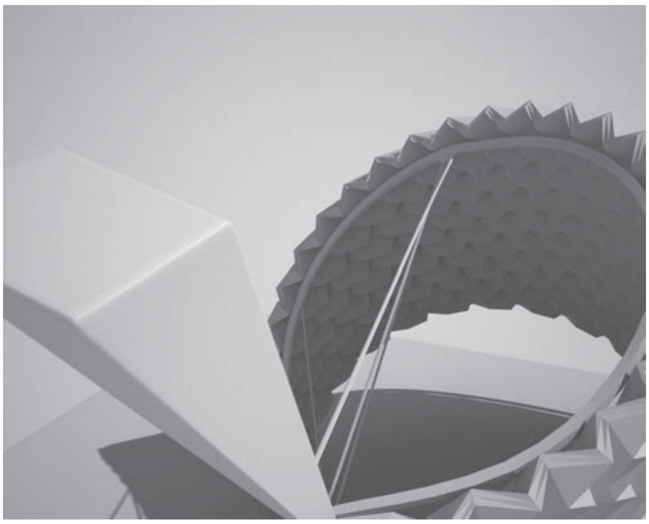

Figure 6. 3D preview of final structure (Source: processed by authors)

In order to transfer diagram into a structure (transition from geometry, which is purely abstract, to material surface, which is physical), one needs to study the system on different levels and understand its behavior and properties affected by geometry, shape, material, structural behavior, etc. That is the reason for implementing analysis and evaluations of structural behavior in this phase of the project. Also, details such as connections and assembling methods were suggested. Finally, the building of small scale prototypes, by the 3D printing technology simulated transposition of the geometries to material constructions.

Geometry Modeling

A constant advancement of the CAD re-emphasizes the importance of geometry, facilitating experimentation with diverse concepts in computational design context, including origami. Construction of 2D diagrams of origami, potentially yielding non-standard grid forms in the plane, offers a new platform for designing innovative mesh configurations and modeling of folded-plate systems.

Generating Folding Pattern

After choosing a folding pattern interesting for further elaboration, the next step was to define reference surface geometry which will represent the final state of free-form folded surface. The surface is mathematically represented as a NURBS surface (Piegl and Tiller, 1997).

Free-form geometry applied in the example is from the class of ruled surfaces. Those surfaces are generally defined by two diretrix NURBS curves $C_1(u)$ and $C_2(u)$ of equal degree, on the same knot vector. These curves guide the motion of a generatrix lines $C_g(v)$. Thus generated ruled surface $S(u,v)$ actually represents a linear interpolation between curves $C_1(u)$ and $C_2(u)$, and could be un-rolled in the plan (Figure 7).

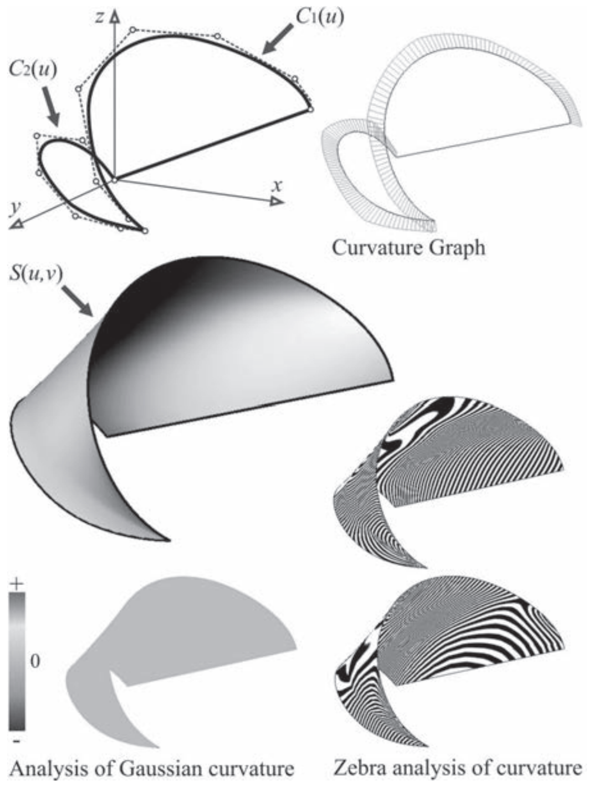

Figure 7. Reference surface geometry (Source: processed by authors)

The generic NURBS surface was the input (reference) geometry introduced to the algorithm which will plot a folding pattern on it. The folding pattern was previously selected after running the generator through a sequential process of string and folding pattern generation iterations, between all diverse possible folding patterns of that generator.

The algorithm includes the following steps:

1. A subdivision of the reference surface domain by a grid of interpolation curves. The density of curves is based on the resolution of desired folded form. The grid of the interpolated curves should match the grid of hinge-lines of folded surface. Based on the selected folding pattern inner and outer hinge-lines should be differentiated.
2. Offsetting inner hinge-lines and adjacent points.
3. Generation of a series of points in association with the reference surface at the intersection of hinge-lines as basic generative geometries.
4. Based on the selected folding pattern connection of points with straight line segments – edges of triangular plates.
5. Generation of triangular plates of folded surfaces. (Figure 8)

We considered important design parameters of folded-plate systems, such as the angle between adjacent plates and the size of the plates. These parameters determine the overall depth, configuration, and consequently, structural behavior of the system. A compromise between practicality and structural efficiency are angles in the order of 70-110 degrees. Increasing angles will cause an insufficient stiffness of the system. Additionally, the approximate depth to span ratio for simply supported system ranges from L/8 to L/15 for concrete system, and L/7 to L/12 for plywood folded plates. Pre-stressed concrete system can be more slender. Parametric linking of the folded surface with hinge-lines enables the modifications of the folded surface by the manipulation of hinge-lines (length, shape, DOF). Changing the depths of creases or their frequencies allows the exploration of structural behavior and spatial qualities.

Small Scale Prototyping

Besides standard presentation methods, a rapid prototyping technology was applied for the production of the 3D physical models. Rapid prototyping (RP) is a term used to denote technologies, conditionally speaking, for fast production of the 3D physical models based on geometry defined by the CAD software (Dimitrov *et al*., 2006). Due to its advantages, the technology initially developed for industrial production found its application in different fields. In architecture, the RR is suitable for the visualization and testing of ideas in different stages of design, and as a step preceding digital production of large scale prototypes and real structures. The assumption is that in the future digital production technologies will have wider application in the building industry (Khoshnevis, 2004).

Prototyping Technology

All RP techniques have in common that physical models are produced progressively by addition of thin layers of material, in the process referred to as the Additive Fabrication (AF) or Layer Manufacturing (LM). Currently available technologies greatly differ depending on the time required for the model production, properties of the applied material, production costs, finishing quality, etc. (Dimitrov *et al*., 2006). The selection of adequate technology depends mainly upon the purpose of the model. In this case we applied the Selective Laser Sintering (SLS) technology (Deckard, 1986). The reason for the selection of this technology was its facility for making very complex geometries directly from the CAD data. The SLS uses a laser as its power source to sinter powdered material, and create a solid structure, i.e. mass that has a desired 3D shape. In comparison with traditional methods of construction of architectural mock-ups, the advantage of the RP is unquestionable.

Construction of Small Scale Prototype

The construction of the prototype included following goals:

1. To communicate and evaluate design concepts efficiently;
2. To demonstrate feasibility of structure based on the purposed geometry; and
3. To get some first-hand experience with the prototyping method.

Two main considerations had to be taken into account: material properties and limitations and process limitations.

For the production of the prototype we used the Spectrum Z™510 device, from the class of high precision functional modelers. This printer has the following properties: (1) build speed 2-4 layers per minute; (2) build seize 254X x 356Y x 203Z mm; (3) material – high performance composite; (4) layer thickness 0.89-2.03mm; (5) resolution 600 x 540 dpi. The device is supported by a system software that accepts models in the STL, VRML and PLY file formats as input (Figure 9).

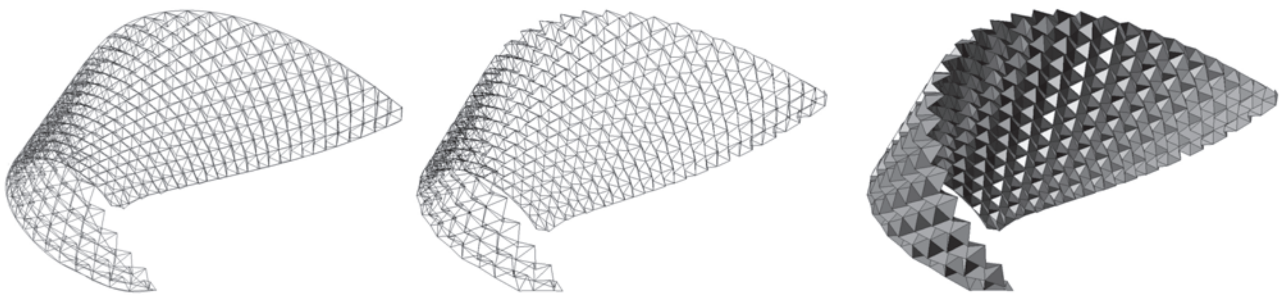

Figure 8. Generation of complex folding structure
(Source: processed by authors)

Figure 9. ZPrinter®510- device used for the fabrication of the model (Source: authors)

The process included following steps:

1. Model preparation included the conversion of the 3D solid model of folded-plate structure generated in the CAD software in the STL file format. The folded-plate was assigned a standard wall thickness of 2mm.
2. The model was sliced into 2D parallel layers of the standard 0.1mm thickness.
3. Transfer of the model to the machine for calculating a build path, i.e. the most efficient way to print the model.
4. Transfer of information about each layer to the processing head of the machine, and the model production by adding material in a layer-by-layer fashion. The laser selectively fused powder material by scanning cross-sections generated from the 3D digital model on the surface of a powder bed.
5. After each cross-section was scanned, the powder bed was lowered by one layer thickness, and a new layer of material was applied on top. The process was repeated until the model was complete.

During the construction, the model was surrounded by un-sintered powder, not requiring supporting structure, which facilitated the fabrication of complex geometry. Supporting arches, pylon and rod ties were made of steel and painted (Figure 10).

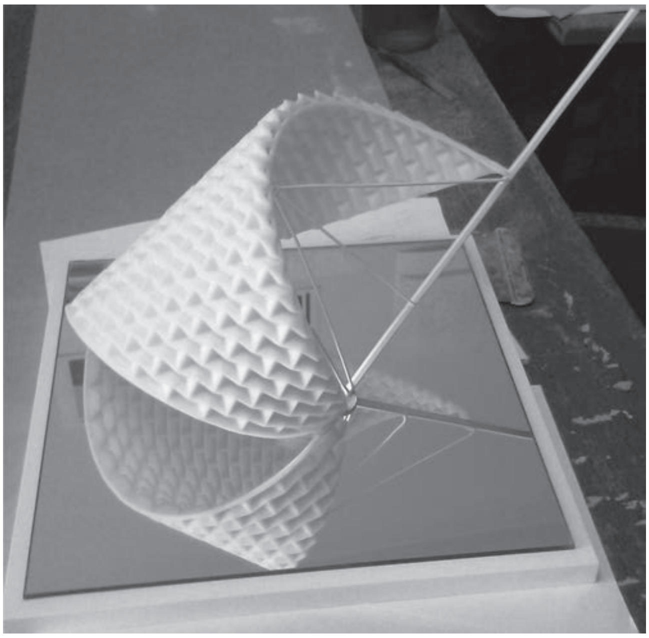

Figure 10. 3D printed small scale prototype (Source: authors)

CONCLUSION

Revising origami as a medium of exploration and consideration of origami diagrams as a meta-language for the development of visual algorithms of design patterns and forms could result in a new cognitive experience. As illustrated by this research, origami diagrams have potential to provide meshes of structural forms. Augmented into the 3D space, they generate classes of structures and structural elements with a proper structural stability. Diagrammatic relations might be regarded as a guarantee of structural order and stability. Further on, patterns developed in the diagrams can be experienced in the virtual or physical medium. On the other hand, not all forms are suitable, so careful analysis is usually required to satisfy all spatial, formal and structural needs. However, it is possible to state that the idea of the implementation of origami design methodology facilitates architects to extend the vocabulary of structural forms.

The presented examples have illustrated that origami can be considered as a prospective design exploration tool that enables formal and structural studies in constrained design conditions and the advancement of the design process. At the end of the course, as it was anticipated, students became aware of the geometry of folds as a source of inspiration. They developed new areas of competence with regard to structural design methodology, and exceeded themselves in terms of interesting designs. Our future expectation is that we will build upon our experience with a multidisciplinary, integrated design approach and see exciting results in terms of innovative project proposals. In that respect the research is left open-ended.

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